Lab Report Outline—the Bones of the Story

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Section: 6

TITLE: Using Hooke's Law to determine the behavior of an elastic cord

ABSTRACT:

In this experiment, we aim to characterize the behavior of an elastic cord using Hooke's law. To achieve this goal, we analyze the force-displacement relationship of a system consisting of a mass hanging vertically from an elastic cord at rest. First, we observe the linear relationship between the weight (mg) and displacement to find the spring constant at multiple lengths of the cord. We then determine the relationship between the length of the cord and the spring constant. We find that as we increase the length, k decreases in a proportional fashion. The linear relationship between force and displacement and the inverse relationship we find between length and spring constant allow us to characterize the behavior of the elastic in a way that allows us to predict how the elastic would behave at much greater lengths.

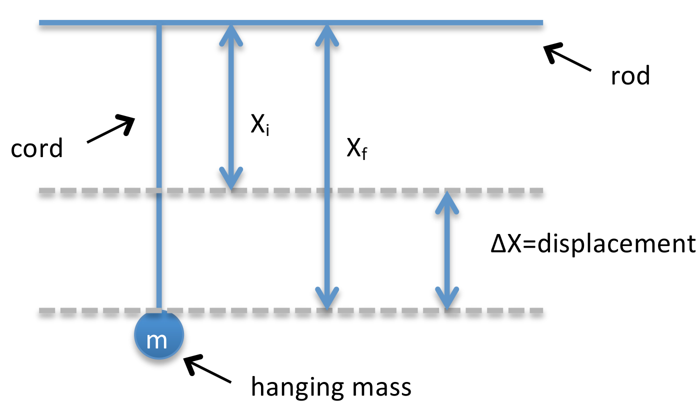
INTRODUCTION:

The goal of this experiment is to characterize the behavior of an elastic latex cord using Hooke's law, $\vec{F}_{spring} = -kx$. Our system consists of a mass vertically hanging from the elastic cord at rest. Because acceleration is zero and the only two forces acting in our system are the force of the spring and the opposing gravitational force (F=mg), we can come up with a combined equation, $mg = kx$. This allows us to examine the relationship between weight (mg) and displacement (x) to find the spring constant (k), assuming that the elastic behaves similarly to that of an ideal spring, and can thus be modeled linearly. We also want to determine how the length of the cord affects its force-displacement relationship. We hypothesize that as the length increases, the spring constant will decrease.

METHODS:

Our method is to observe the displacement (position when stretched – unstretched position) as we vary the mass. Once we plot weight vs. displacement from 10 different masses, the slope of the line gives us the value of the spring constant, k. We carried out this method at 5 different cord lengths in order to determine how k changes with the length of the cord.

Diagram:



Setup/procedure:

1) Tie two slip knots in cord and attach one knot to the rod, letting the other hang down so that it rests vertically in the air
2) For each cord length:
   a) Measure unstretched length of cord (X_i) from the top of the knot attached to the rod to the top of the knot that is hanging in space
   b) Attach first (smallest) mass to the hanging knot and measure the stretched length (X_f) from the top of the knot attached to the rod to the top of the knot attached to the mass
   c) Subtract the unstretched length (X_i) from the stretched length (X_f) to get the displacement (ΔX)
   d) Repeat for 10 different masses
   e) Plot weight (mg) vs. displacement and find the slope, which represents the spring constant, k
3) Undo slip knot attached to rod and make a new slip knot at a different length on the cord and repeat a) through d)
4) Repeat this for 5 different cord lengths

RESULTS:

Table 1: Weight vs. Displacement

X_i (m)		0.820	0.696	0.490	0.385	0.215
Mass (kg) ± .01	**Weight** (N)	**Displacement** (m) ± .001	**Displacement** (m) ± .001	**Displacement** (m) ± .001	**Displacement** (m) ± .001	**Displacement** (m) ± .001
0.05	0.4905	0.178	0.147	0.115	0.080	0.047
0.07	0.6867	0.259	0.219	0.175	0.125	0.070
0.09	0.8829	0.390	0.327	0.245	0.183	0.100
0.11	1.0791	0.545	0.444	0.337	0.255	0.140
0.12	1.1772	0.622	0.519	0.385	0.293	0.162
0.13	1.2753	0.712	0.594	0.430	0.335	0.185
0.14	1.3734	0.805	0.664	0.480	0.377	0.208
0.15	1.4715	0.902	0.739	0.520	0.423	0.232
0.16	1.5696	0.995	0.819	0.570	0.468	0.255
0.17	1.6677	1.100	0.899	0.620	0.512	0.280

Table 1 indicates the mass, weight (mass*gravity), and the corresponding displacement for five different cord lengths. The first row of the table specifies the initial unstretched length of the cord.

Figure 1: Force-Displacement Relationship

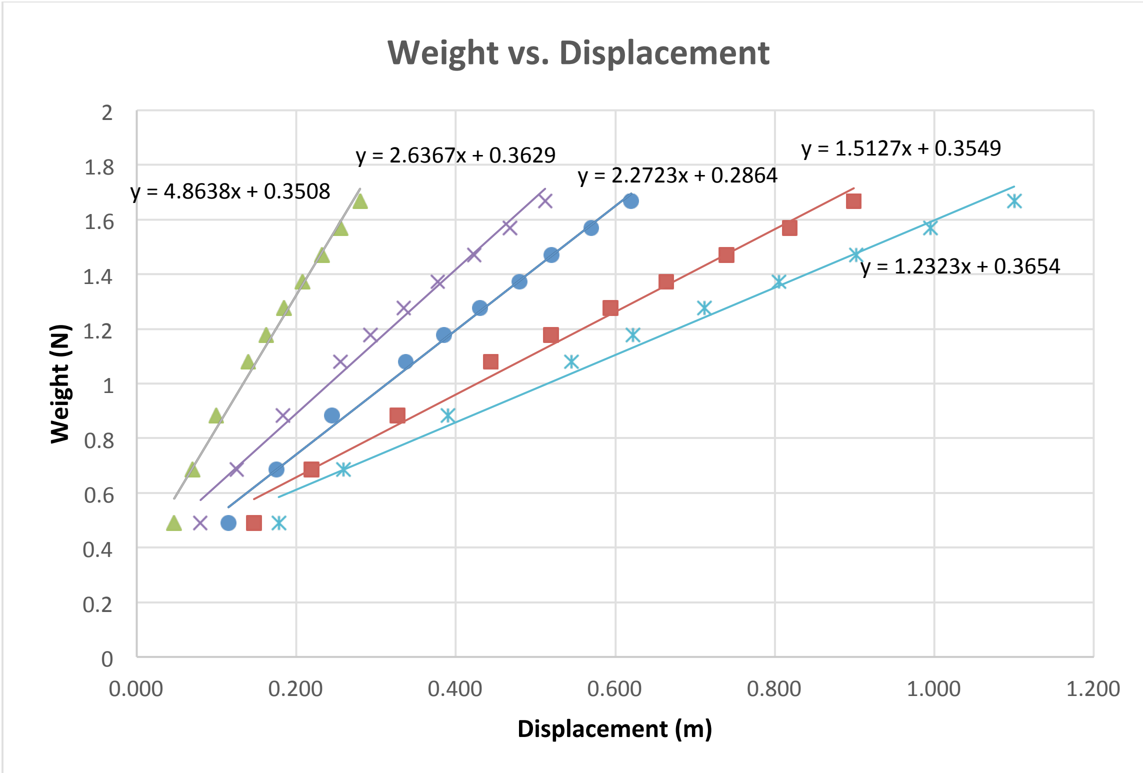


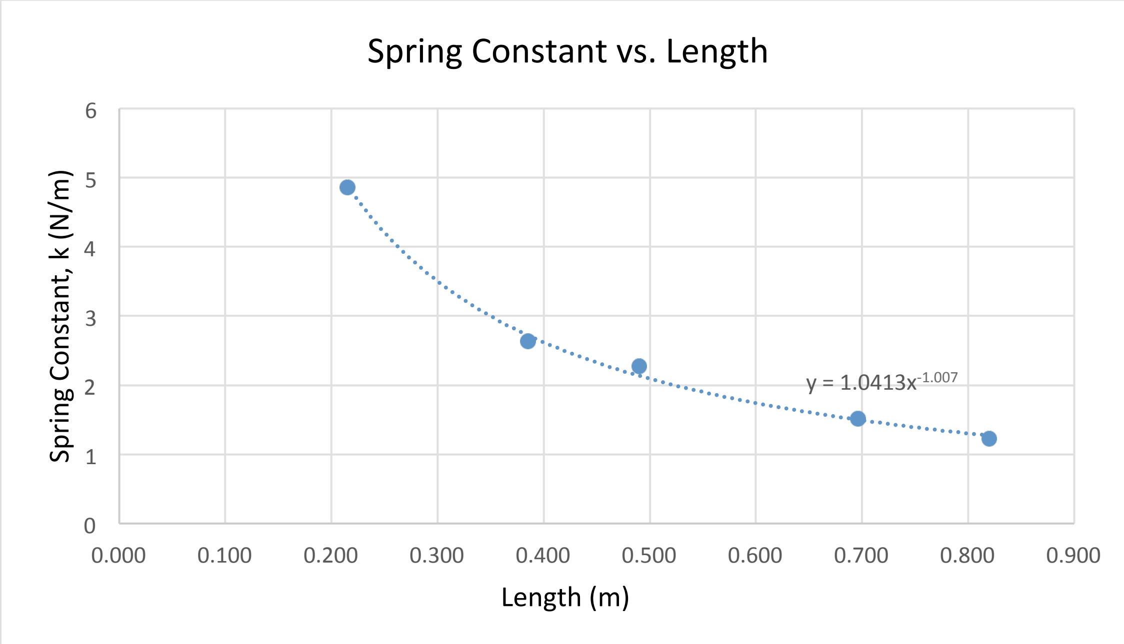
Figure 1 shows the weight vs. displacement curves for each different cord length. Each trendline has an equation that resembles mg=kx. The slope (coefficient) of each line represents the spring coefficient, k for that cord length.

Table 2: Spring constant (k) vs. length

k (N/m)	**length** (m) ±.001
1.2323	0.820
1.5127	0.696
2.2723	0.490
2.6367	0.385
4.8638	0.215

The values of k in the table above are taken from the slope values found in Figure 1. The length corresponds to the initial (unstretched) length of the cord.

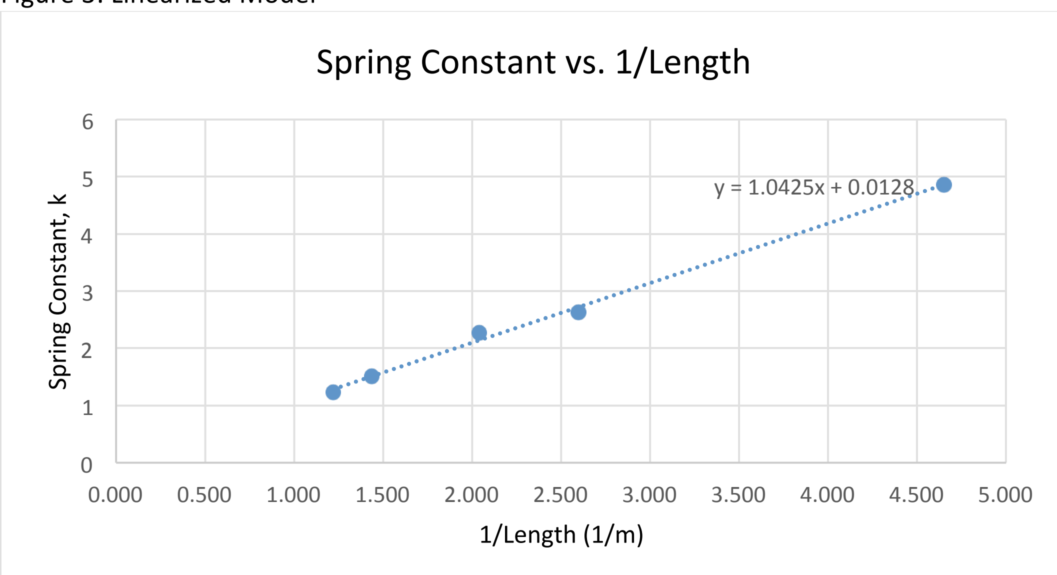
Figure 2: Curve-fit Model



Curve-fit equation: $k = 1.0413l^{-1.007}$

This equation models the power relationship between the spring constant and the length of the cord. Because the power ~-1, the relationship between k and length can be thought of as an inverse.

Figure 3: Linearized Model



Linearized equation: $k = \frac{1.04}{l} + 0.013$

Figure 3 models the linearized relationship between the spring constant and length. We find a very clear linear relationship between the spring constant, k and the inverse of the length (1/l).

Excel regression analysis:

	Coefficients	*Standard Error*
Intercept	0.0128	0.0930
X Variable 1	1.0425	0.0346

uncertainty for slope (std. error)=0.03 % uncert = 3%

uncertainty for y-intercept= 0.093 % uncert= 700% (st. error>coeff.)

Our results give us very useful information about the behavior of the elastic. We find that the relationship between force (mg) and displacement is linear when the length of the cord is held constant. We also find that the spring constant is inversely related to the length of the cord.

DISCUSSION:

Because the goal of this experiment was to determine the behavior of the elastic cord, there were no theoretical results that we could compare our coefficient values to. That being said, we learned meaningful information about the elastic's behavior.

Sources of uncertainty in our experiment mainly arise when measuring the displacement of our cord. Although the raw uncertainty in the displacement is small (.001 m) indicating a low level of uncertainty, it's very possible that the human eye cannot detect measurements this small. Additionally, these measurements can be thrown off if you are not at eye level with the hanging mass, which is not always possible if the mass is too high up. Additionally, although we tried to measure from the same point (the top of the slip knot attached to the hanging mass) each time, this may not have been the exact point from which the displacement was measured in every trial.

Our results indicate that the force-displacement relationship can be modelled linearly when the initial, unstretched length of the cord is held constant. Our results support our hypothesis that when the unstretched length changes, k decreases. Because our coefficient in our linearized model is ~1 with a small percent uncertainty, we can deduce the following relationship between k and length: k=1/l.

CONCLUSION:

We found that we can in fact model the relationship between the force of gravity and displacement as a linear relationship using the equation mg=kx. We also found that as we varied the length of the cord, k decreased accordingly. We found that there is a direct inverse relationship between length and spring constant. This information is necessary to the execution of the next steps of the Bungee Challenge. These conclusions imply that we can continue to model the elastic as we do an ideal spring. Increasing the length of the cord is similar to attaching two springs with the same spring constant, which thereby decreases their combined spring constants.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Isabella Swanson