Lab Report Outline—Bungee 1 Article Assignment

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TITLE: Is an elastic cord's displacement linear enough to represent it as a spring?

ABSTRACT:
An elastic cord is being used in a critical bungee jump experiment. Therefore, it is extremely important to consider how the cord will behave when a mass attached to one end is dropped. Will the elastic behave like a spring as stated in Hooke's law? Hooke's law states that $F_{total} = -kx$, or the total force on the system is equal to the negative k constant (or spring constant) times the displacement of the spring. In order to answer this question, we created an experiment that changed the amount of mass hanging from the end of the cord to see the effect on the static displacement of the mass. We then graphed and analyzed the linear relationship between the total force on the elastic, which is the weight of the mass hanging from it, and the displacement of the elastic. This showed that F_{total} = 1.467*x* + 0.596. The coefficient of *x* is considered the k constant, in relationship to Hooke's force law. Overall, this appears to show the action of the cord is somewhat linear to a spring by a "k" constant of 1.467, but the y-intercept of 0.596 must be considered when calculating the displacement of this elastic at this constant length 0.510 m (+/- 0.005 m). Our calculated *k* value has a 30% error when compared to the equivalent accepted *k* value found from Hooke's law, which is much greater than our uncertainty in k_{calc} (6%), so our equation for the behavior of the spring at this length is not accurate to Hooke's law. Ultimately, the behavior of our elastic cord cannot be characterized as an ideal spring by Hooke's Law.

INTRODUCTION:

An elastic cord will be used in a bungee experiment and it is essential to know if the elastic cord acts similarly to a spring. If the elastic cord is linear enough to do so, then the bungee jump can be created and calculated in a similar fashion to a spring. The cord's functional relationship between force and displacement must be determined for a successful bungee experiment.

Relevant equations:

- Hooke's Law: $\vec{F}_{spring} = -k\vec{x}$
    - k: Spring constant, the strength of the spring, the number of newtons of force needed to stretch the spring one meter.
    - $\vec{F}_{spring}$: spring force, which is negative because the force of the spring is a restoring force, pulling back to the un-stretched position.
    - $\vec{x}$: stretch or displacement of the spring
- Stretch/displacement of the elastic: Displacement= $x - x_L$
    - x_L: unstretched length of the elastic (no mass)
    - x: length of elastic with mass hanging on end

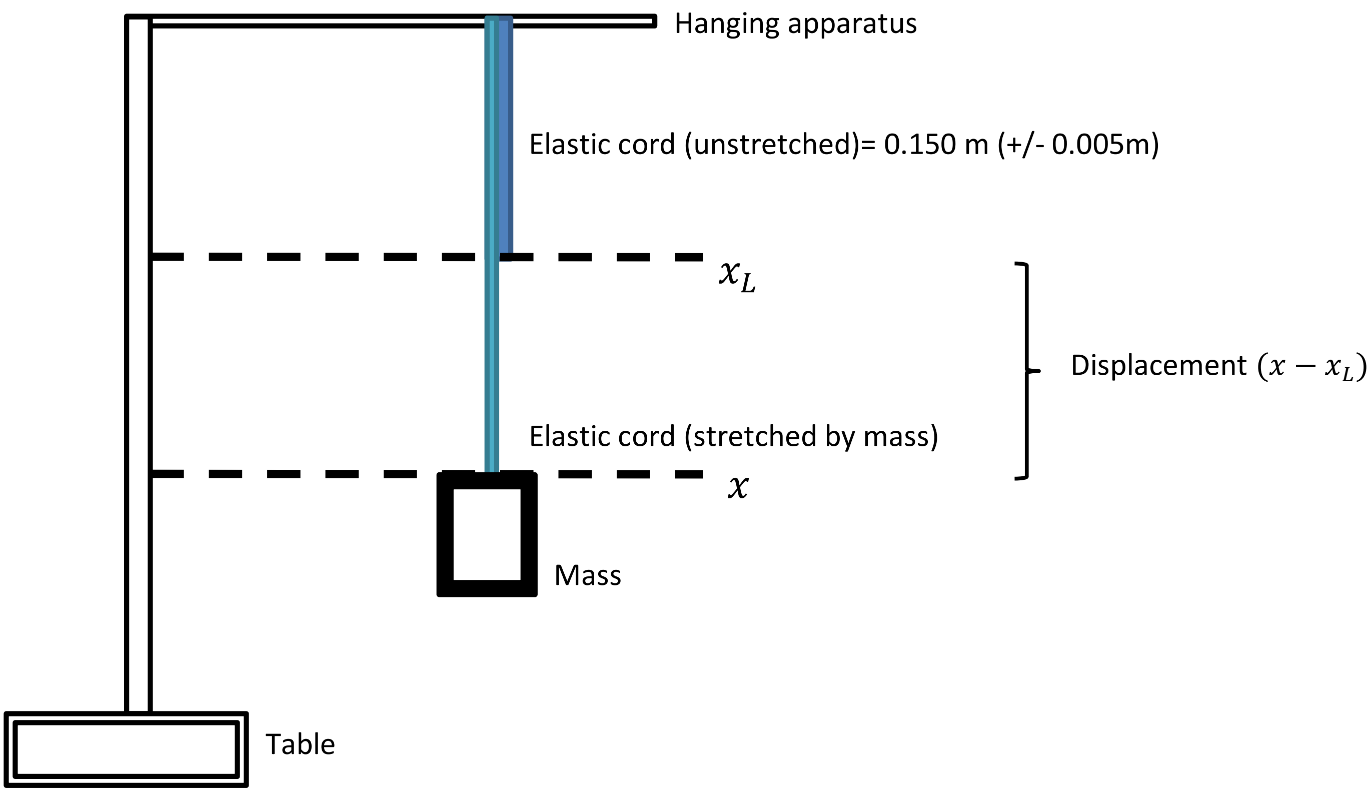
Hooke's Law shows the linear, and therefore proportional, relationship between the spring force and displacement of a spring. This may be able to relate to an elastic cord because elastic material is made of long-chain molecules that stretch and deform (potentially similar to a spring being stretched), and recover when the force is released. While some elastics can stretch multiple times their original length without damage, others change over time and the bonds change. Therefore, the relationship between the force on the elastic cord and the displacement of the cord will be compared to Hooke's law to see if our elastic cord has long-chain molecules that deform and recover similarly to a spring.

Hypothesis: I expect the elastic cord to be linear to Hooke's law due to the ability of rubber molecules to recover after being stretched, similar to a spring being stretched and then returning to equilibrium when released.

METHODS:

In order to determine if the elastic cord has a linear relationship between force and displacement, we hang different masses from the same length of unstretched elastic and find the displacement ($x - x_L$) length. This allows us to see the relationship between the weight of the system (mass times acceleration due to gravity) and the displacement of the elastic cord, which will be graphed in order to see if the relationship is linear and can be modeled with Hooke's law ($\vec{F}_{spring} = -k\vec{x}$), with the slope as the *k* constant.

Figure 1: Elastic Cord Displacement Set-Up. Length of hanging cord unstretched is measured, then mass is added to find the displacement of the cord.



Setup:

- A metal hanging apparatus was set up on the edge of a table.
- An elastic cord was tied to the overhanging hanging apparatus so the length of the unstretched elastic cord was 0.510 m (+/- 0.005m).
- Small knot tied at the end of elastic to hang mass on.

Procedure:

- Set-up hanging apparatus and elastic cord as described in "Setup".
- Measure length of unstretched cord, X_L, in meters, from bottom of hanging apparatus rod to knot in bottom of elastic.
- Add specified mass to the knot at the end of elastic.
- Release mass to hang at equilibrium.
- Quickly measure length of elastic cord (X), now stretched by the mass (in meters), from the bottom of hanging apparatus rod to knot in bottom of elastic.

- Remove the mass from the elastic to avoid fatigue of the cord. This could result in a change in the recovery length of the elastic cord.
- Repeat procedure for the following masses: 0.05 kg, 0.1 kg, 0.15 kg, 0.2 kg, 0.25 kg, and 0.3 kg. Record length of X (in meters) for each.

RESULTS:

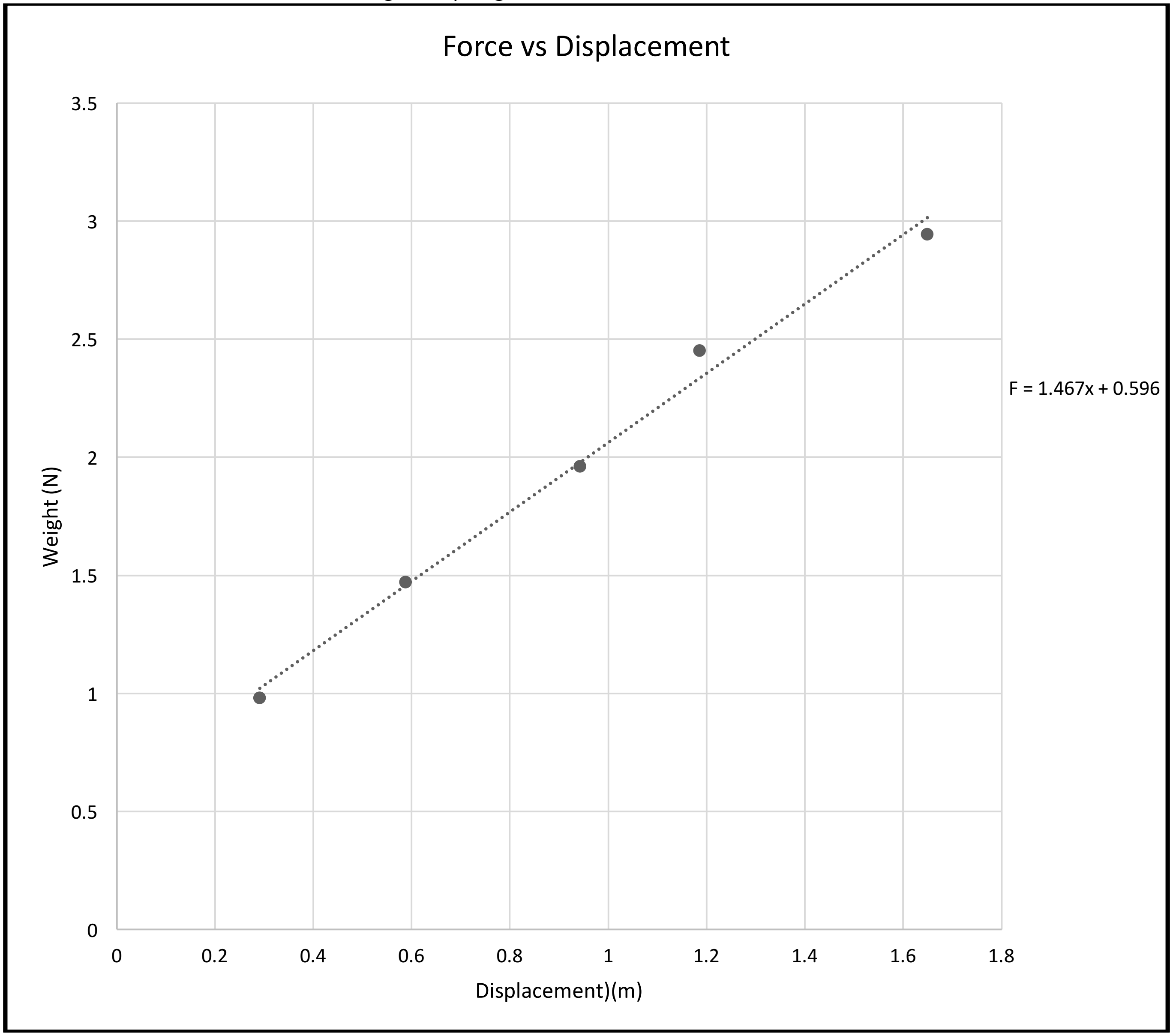
The following data was collected by calculating the displacement distance ($x - x_L$) and the weight ($W = mg$) of each mass on the elastic. The results will be graphed to determine if the cord's relationship between force and displacement is linear enough to be modeled by Hooke's law.

Figure 2: Displacement of an Elastic Cord. For each weight, the displacement of the elastic cord was calculated by subtracting the unstretched length (X_L) from the stretched length (X). The weight was calculated through the formula ($W = mg$), where m is the hanging mass and g is acceleration due to gravity (9.81 m/s^2).

Weight (N) (+/- 0.001 kgm/s^2)	**Displacement (m) (+/- 0.005 m)**
0.491	0.114
0.981	0.291
1.472	0.588
1.962	0.942
2.453	1.186
2.943	1.649

The uncertainty found in the weight is due to the uncertainty of the mass due to the least count of the electronic scale. The uncertainty found in the displacement is due to the raw displacement of X_L and X, with X_L= 0.005m and X= 0.002m.

Figure 3: Force vs Displacement of Elastic Cord. The force acting on this system is the weight of the mass hanging from the cord ($w = mg$), and the displacement is ($x - x_L$). The slope, 1.467 (N/m), is the "k" constant of this elastic cord acting in a spring-like manner.



The first data point was removed from the graph because it did not fit with the trend line.

Equation of curve-fit: F=1.467x + 0.596

Regression analysis of linear fit in Figure 3:
- uncertainty for slope=0.079 % uncert= 6%
- uncertainty for y-intercept= 0.083 % uncert= 14%

Value of interest: *k*= 1.467 N/m
- This is a value of interest because it is a constant in a proportional equation between force and displacement of the elastic cord, very similar to Hooke's force law where the force of the spring is proportional to the displacement of the spring.
- Uncertainty of experimental value: 0.079 % uncert: 6%
- Uncertainty was found through *Excel* regression analysis of the linear fit of Figure 3.

Value of interest: y-intercept= 0.596

- This is a value of interest because the force and displacement of the elastic cord are not completely linear unless you consider this y-intercept as well.
- uncertainty for y-intercept= 0.083 % uncert= 14%
- Uncertainty was found through *Excel* regression analysis of the linear fit of Figure 3.

The relationship between the displacement of the elastic band and the force on the band (weight) is mostly directly proportional, as shown by the equation of the cure-fit F=1.467x + 0.596, which is relevant to the purpose because Hooke's law says that the force of the spring is proportional to the displacement, $F_{spring} = -k\vec{x}$.

DISCUSSION:

In order to quantitatively compare our equation of the curve-fit for the elastic cord to Hooke's law, our experimental value of *k* (1.467N/m +/- 6%) will be compared to a calculated value of *k* in Hooke's law to find the percent error. This calculated value, or $k_{accepted}$ is found by taking a chosen displacement value from our equation, for example 1.000m, and the force from our equation at that value, for example -2.063 kgm/s^2, and solve for $k_{accepted}$ in Hooke's law. The percent error will then be compared to our percent uncertainty of k_{calc} to determine how accurate our equation for the behavior of the elastic is to Hooke's force law for springs.

Experimental Value: k_{calc} = 1.467 N/m

Uncertainty of Experimental Value: 0.079 N/m

Accepted Value of Hooke's Law: $k_{accepted}$ = 2.063 N/m

Percent Error: 30%

Percent Uncertainty: 6%

Our curve-fit for the behavior of an elastic cord with the slope k_{calc} is not accurate with the value of $k_{accepted}$ in Hooke's law. Therefore, Hooke's law may not be used to describe the behavior of this elastic cord as a spring.

Sources of uncertainty:

- Measure of length of elastic cord: The meter stick has a least count of (+/- .001 m) and the length of the elastic cord had to be recorded quickly in order to reduce the mass' impact on stretching the cord. Therefore, the uncertainty was (+/- 0.005 m) due to the speed and quality with which the length could be recorded.
- Possibility that the elastic cord stretched irreversibly as more mass was added: This would result in a longer starting length (X_L), and would therefore affect the *k* constant (if applicable) since each length of the cord has a different spring constant due to the number of long-chain molecules that deform and recover, and the displacement measured for the proceeding measurements since the starting length would be different.

My hypothesis is not supported by my main results. At the unstretched length of 0.510 m (+/- 0.005), the force on the elastic cord was mostly proportional to the displacement of the elastic by a constant of 1.467 (+/- 0.079). However, the relationship is not COMPLETELY proportional since the y-intercept is not at 0 m. This proves an important factor because the percent error in the k_{calc} value compared to $k_{accepted}$ from Hooke's law at the same point was much greater than the uncertainty of k_{calc}.

CONCLUSION:

The relationship between the force and the displacement of the elastic cord does not behave in a linear relationship comparable to the behavior of springs. The force and displacement is mostly proportional with the constant, *k*= 1.467 (+/- 0.079) at the unstretched length of 0.510 m (+/- 0.005 m) however the y-intercept of 0.596 shows there is some discrepancy between the force of springs and the force of this elastic and that Hooke's Law is not applicable to the elastic cord at this length.

These conclusions about the behavior of the elastic cord are important to consider for the bungee jump. Since the cord does not strictly follows Hooke's law in its relationship between force and displacement, extra steps will be necessary in preparation of the bungee jump. This linear equation $F=1.467x + 0.596$ can be used as a rough estimate for the proper length of the cord, but consideration must be taken that there will be an increase in the stretch of the elastic cord as it gets longer, and how different this elastic cord acts compared to a spring.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.