Lab Report Outline

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TITLE:

- Modeling the behavior of a bungee cord using Hooke's Law

ABSTRACT:

- We were tasked with giving an egg on a bungee cord the most thrilling ride possible when dropped from the 4th floor balcony of the Great Hall in the Science Center. To begin this process, we decided to attempt to model the bungee cord's actions using Hooke's Law. Hooke's Law states that the force (F) required to compress or extend a spring some distance (X) is proportional by some value (K) to that distance (F = -kx). This experiment investigated these concepts by treating the bungee cord as an ideal spring and examining how it reacted in a static case. The bungee was hung vertically, and a mass was connected to the bottom of the cord. In order to vary the weight force on the "spring", the mass at the bottom was varied throughout experimentation. The displacement of the cord from its normal equilibrium position with no mass attached to its equilibrium position with mass attached was measured using a tape measure and recorded for each different mass. The data was then graphed and both a polynomial and a linear equation were applied to the data. After analysis and for simplicity's sake, the linear equation which was similar to Hooke's Law with a slight modification was determined to be possibly useful for future predictions concerning the behavior of the bungee cord. It was also determined that the graph was slightly concave down which demonstrated that as the mass increased, the bungee cord proportionally stretched more. This means that as the weight force on the bungee increased, the k value was not constant as in an ideal spring but was in fact slightly reduced. Overall, this experiment provided valuable insight into the mechanics and behavior of the bungee cord as well as a potentially useful modified Hooke's Law equation for the bungee cord.

INTRODUCTION:

Purpose or question:

- This experiment was conducted to determine if the bungee cord's mechanics could be modeled using the physics for a vertically hanging ideal spring
- Specifically, we focused on whether the bungee cord could be effectively modeled using Hooke's Law (F = -kx).
  - To do this we needed to calculate a k value for the bungee
  - Also we needed to determine if the bungee's k value would remain constant during experimentation
    - A static experiment was designed in which the length of the bungee was held constant while the force on the bungee was varied and the displacement was measured
- If the k-value remained constant during experimentation, then it would be potentially viable to model the bungee's actions by using Hooke's Law (F = -kx).
- All of this was in order to complete the Bungee Challenge. We were tasked with determining the best way to model the behavior of the bungee cord, so that we could give an egg the most thrilling ride when dropped in the Science Center's Great Hall

Relevant equation:

- $F_{spring} = -kX \rightarrow \mathbf{Mg = -k(X_L - X_o)}$
  - $F_{spring} = W = Mg$
  - $-kX = -k(X_L - X_o)$
- W = weight of the mass attach to the bottom of the bungee cord
- M = mass attached to the bottom of the bungee cord
- k – proportionality constant of the bungee
- X_o – equilibrium length of the bungee cord with mass connected to the bottom

- X_L – unstretched length of the bungee cord

Theoretical Background:

- The system experiences a downward force due to the weight of the mass attached to bottom of the bungee cord, and then stretches a certain distance downward dependent on the magnitude of the weight force and the characteristics of the bungee (stiffness, strength, etc.)
- F_{spring} – the total force on the spring, in this system it is the weight force downward from the masses attached to the bottom of the bungee cord, calculated by multiplying mass with the acceleration due to gravity ($g=9.81\ m/s^2$), measured in Newtons
- -k – proportionality constant, characteristic of the spring, negative because the spring force is a restoring force, in this system we are attempting to determine if this proportionality constant for the bungee cord remains the same regardless of the force on the system, measured in Newtons per meter
- X – the displacement distance from the spring's equilibrium position, in this system it is the distance from the bungee's motionless hanging position with no connected mass to where it hangs at rest with each mass attached, calculated by subtracting X_o from X_L, measured in meters
- Theoretically, if the bungee increasingly stretches the same proportional distance as the force on the bungee increases then the bungee behaves in accordance with Hooke's Law
  - The k value can be calculated by dividing the force applied to the system by the distance stretched
  - This will be done by graphing Force vs. Displacement for the data and analyzing the resulting graph

Hypothesis:

- As the mass attached to the bottom of the bungee increases, the weight force will increase and so too will the displacement distance
- We do not think that the bungee cord will have a constant k value, but we do think we will be able to potentially accurately model its behavior using a modified version of Hooke's Law
  - As the force on the system increases, the k value will get reduced because the bungee cord stretches a fair amount

METHODS:

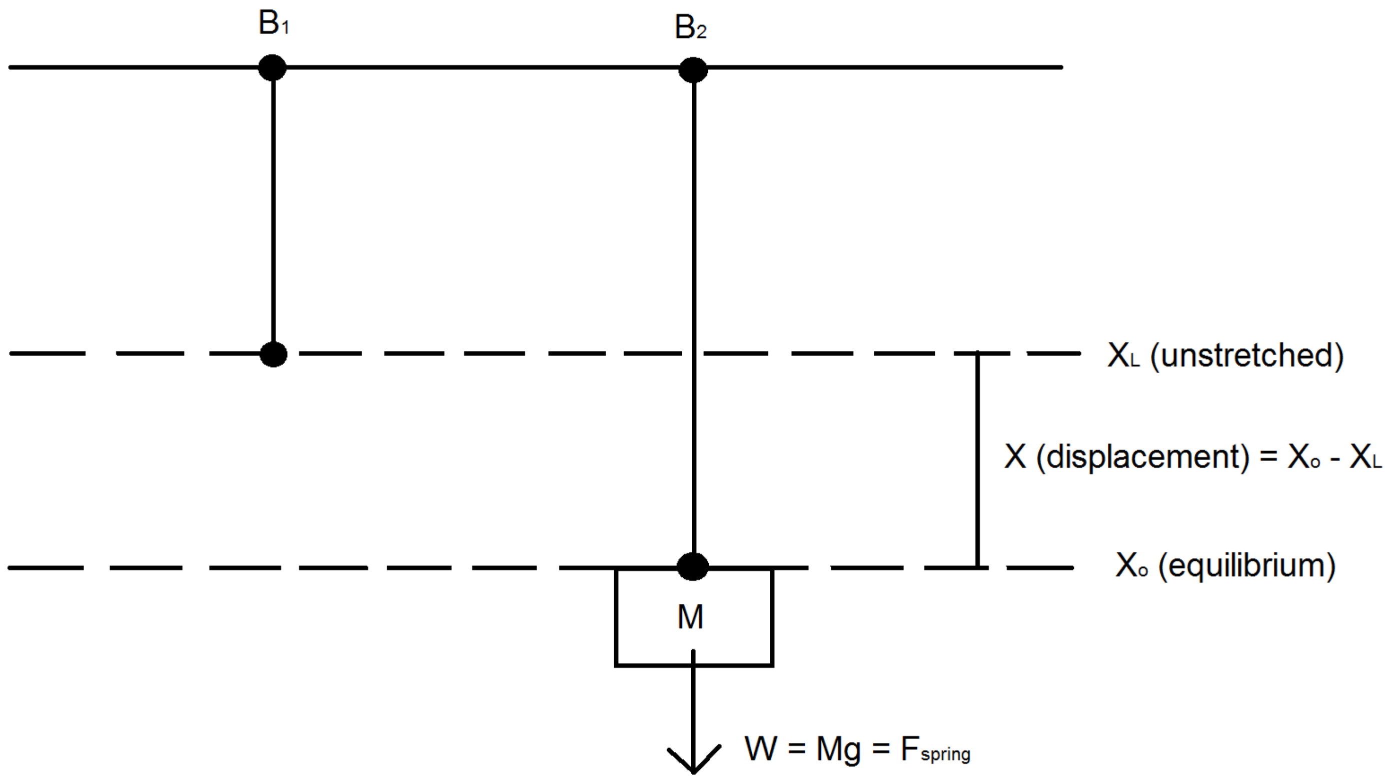
Overall Method:

- To calculate the experimental k value of the bungee cord, we needed to measure the system's total force and its displacement. The total force on the system was varied by varying the mass attached to the bottom of the bungee cord.

Diagram:

Figure #1: Bungee Cord System. Bungee cord at its un-stretched length on the left and bungee cord at equilibrium with a mass attached at the bottom on the right

- B_1 = unstretched bungee cord
- B_2 = bungee cord at equilibrium with a mass attached at the bottom
- M = mass attached to the bottom of the bungee cord
- X_L = unstretched length of the bungee cord, .3 meters
- X_o = equilibrium length of the bungee cord with mass connected to the bottom
- X (displacement) = the recorded data value, calculated by subtracting X_L from X_o
- W = the weight force on the "spring" system, calculated by multiplying mass M by g ($9.81\ m/s^2$)
  - Bungee – thin, black cord about 5 meters or so in length, relatively stretchy and resilient
  - Mass – standard laboratory masses placed on a .05 kg hanger (thin metal platform on which masses can be loaded with an extended hook up top)



Setup:

- Bungee cord was stretched out a couple times prior to experimental use to best prevent permanent cord extension during experimentation
- Two small loops were formed using balloon tying knots on one end of the cord about .3 meters apart so it would properly hang and mass could be attached
- The bungee cord and a tape measure were hung next to each other on lab apparatus protruding over the edge of the table. There was ample room between the bottom of the cord and the ground
- Did not leave mass hanging on the bungee longer than needed to prevent excess stretching of the cord

Procedure:

- The unstretched length of the hanging bungee cord was measured from the top knot to the bottom knot
- Attached mass to the bottom of the cord and released it waiting for the system to reach its new motionless equilibrium position
- Recorded the difference between the new equilibrium length and the unstretched length
- Mass was then added to the bottom of the cord in .02 kg increments all the way up to .25 kg and the displacement was recorded for each mass

RESULTS:

Intro to Results:

- The displacement data was recorded using the tape measure positioned next to the bungee cord system
- The data was then inserted into an Excel graphing template which created a graph of Force vs. Displacement
  - Both a polynomial curve and a linear trendline were fitted to the data on the graph
- The graph's shape was also analyzed to determine if the k value remained constant during experimentation
- Experimental values were obtained from the linear trendline and analyzed

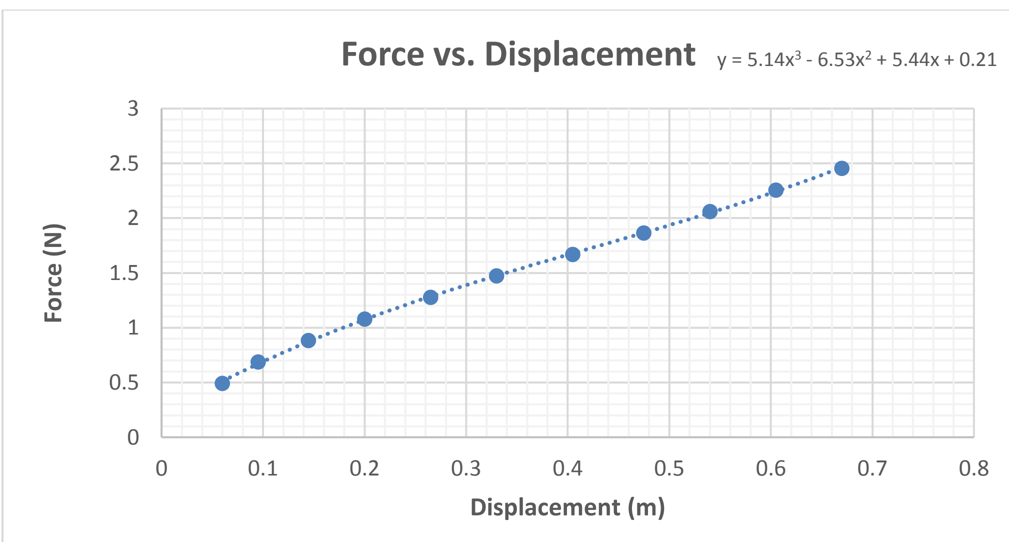
Table:

- **Figure #2: Displacement of the Bungee with Different Masses.** For each mass, the displacement in the bungee length was measured. Force was calculated by multiplying each mass by g (9.81 m/s^2). Uncertainty for the mass and the displacement were estimated. Uncertainty for the force was derived by combining the uncertainty for mass and the product formula

Mass **M** (kg) ($\pm$ .002 kg)	Force **W = Mg** (N) ($\pm$ .02 N)	Displacement $X_L - X_o$ (m) ($\pm$ .005 m)
0.050	0.49	0.063
0.070	0.69	0.094
0.090	0.88	0.145
0.110	1.08	0.200
0.130	1.28	0.266
0.150	1.47	0.329
0.170	1.67	0.405
0.190	1.86	0.477
0.210	2.06	0.542
0.230	2.26	0.604
0.250	2.45	0.670

Graph:

- **Figure #3: Force vs. Displacement of the Bungee System.** A 3[rd] order polynomial equation has been fitted to the data which is slightly concave down

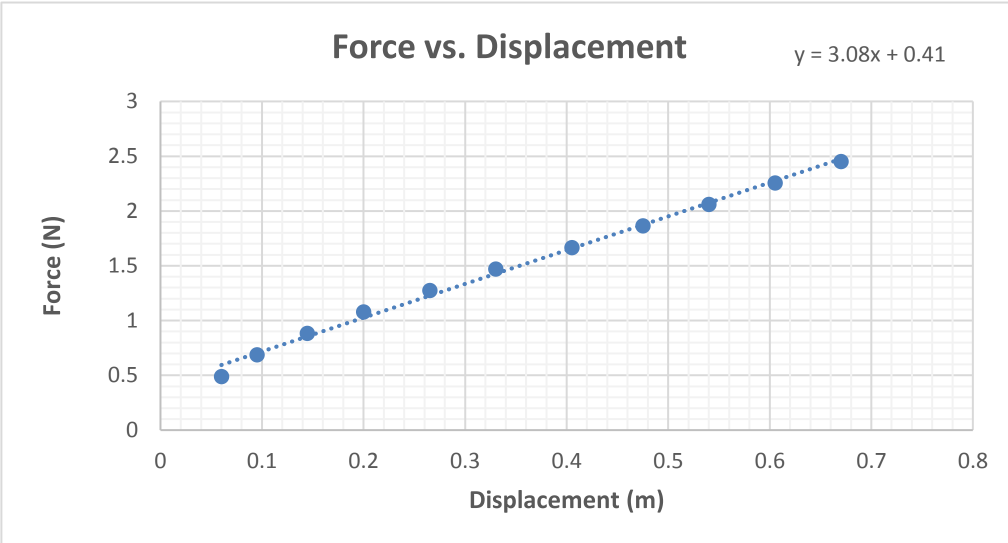


Equation:

- $W = 5.14(X_L - X_o)^3 - 6.53(X_L - X_o)^2 + 5.44(X_L - X_o) + 0.21$

Linear Fit Graph:

- **Figure #4: Force vs. Displacement of the Bungee System.** A linear best fit line has been applied to the same data. The slope of 3.08 ($\pm$ .07) N / m is the experimental k-value for the bungee cord.



Linear equation:

- W = 3.08($X_L - X_o$) + 0.41

***Excel* regression analysis**:

uncertainty for slope= $\pm$ .07 N / m % uncert= 2%

uncertainty for y-intercept= $\pm$ .03 N % uncert= 7%

Propagation-of-uncertainty analysis:

uncertainty in coefficient(s)= $\pm$ .02 m % uncert= 4%

- This uncertainty was derived by using the product formula on the estimated uncertainty for the displacement

Identify experimental values of interest:

- Values were obtained from the equation of the linear trendline fitted to the data specifically the slope and the y-intercept
- These are of interest because they represents the experimental k value for the bungee cord (slope) and a modification to the traditional Hooke's Law equation (y-intercept)

value obtained = 3.08 N / m

uncertainty of experimental value(s) = $\pm$ .07 N / m % uncert = 2%

value obtained = 0.41 N

uncertainty of experimental value = $\pm$ .03 N % uncert = 7%

- Uncertainty in the experimental values was obtained from the linear regression specifically from the outputs of the standard error in both the x-variable and the intercept

Other Pertinent Info:

- The data was not actually linearized but rather a linear line of best fit was fitted to the same data in order to obtain a specific Hooke's Law equation for the bungee albeit one with a modification in the form of a y-intercept
- The data was overall slightly concave down in shape
- The slope of the polynomial curve was slowly getting shallower / flatter as the force increased

Summarize Results:

- An experimental equation, $W = 5.14(X_L - X_o)^3 - 6.53(X_L - X_o)^2 + 5.44(X_L - X_o) + 0.21$, was modeled by fitting a 3rd order polynomial best fit curve to the slightly concave down data. Similarly, a second experimental equation, $W = 3.08(X_L - X_o) + 0.41$, was modeled by fitting a straight trendline to the same data. The slope of this equation, 3.08 N / m, is equivalent to the experimental k value of the bungee cord and the equation itself represents an adapted Hooke's Law model for the bungee. The two best fit equations can now be analyzed to see if the k value is constant throughout experimentation and which potentially should be used to predict the bungee's actions.

DISCUSSION:

Acceptability and Extra Testing:

- The uncertainty appears to be acceptable and appropriate for both the linear and polynomial best fit equations
- Intermediary test for error – a force sensor was connected to the bungee system in which the mass was now to be dropped from the top of the bungee. The bungee's maximum displacement (X_{max}) downward from the equilibrium point with mass attached was recorded using a slow motion camera. Then using our modified Hooke's Law equation, we predicted the force on the bungee from the measured displacement.
    - Measured displacement ($X_{max} - X_o$) = .5 m
    - Calculated force using our equation ($W = 3.08(X_{max} - X_o) + 0.41$) = 1.95 N
    - Measured force from the force sensor = 1.94 N
    - Since the modified Hooke's Law equation almost perfectly predicted the force on the bungee, it passed this intermediary test and certainly seems to be a viable option for modeling the bungee's behavior
- For the final test of our model for the bungee cord, we will be performing the Bungee Challenge in the Science Center Great Hall
    - Using our determined model of the bungee's cord behavior, we will predict the best way to give an egg the most thrilling ride while dropping it from the 4th floor balcony in the Great Hall

Sources of uncertainty:

- The unstretched length of the bungee at the end of experimentation was definitely slightly longer than the measured unstretched length at the beginning due to all the stretching the bungee underwent. This means we might have been slightly off in some of our later displacement measurements
- The bungee system was never truly at resting equilibrium because it was almost always moving slightly up and down when displacement measurements were taken
- The knots / loops in the cord also stretched as well when mass was added which could have slightly thrown off the measured displacements

Any further observations:

- Of the two fits (linear and polynomial), the 3rd order polynomial curve better fits the data collected
    - This was determined by examining the R^2 values for both equations
    - The polynomial curve had a higher R^2 value than the linear trendline meaning it was a better fit to the data

  - However, the adapted Hooke's Law linear equation is also a lot simpler and easier to actually use and manipulate than a 3^{rd} degree polynomial equation
- The data was slightly concave down which demonstrated that as the force increased on the bungee system, the slope was reduced and thus the k value was reduced
  - Therefore the k-value is not constant when the force is varied
  - The bungee will proportionally stretch more when more force is added into the system
- Despite the fact that the true k value of the bungee is not constant during experimentation, the modified Hooke's Law equation derived from the linear trendline can still serve as a potential predictor of the bungee's actions as shown in the intermediary test preformed

Results Support Hypothesis:

- The results were in agreement with my hypothesis. The k value was not constant when the force was varied in the bungee system. Also as expected, when mass was added at the bottom of the bungee cord, the weight force increased and so did the displacement distance. Finally, our modified Hooke's Law equation did an accurate job of modeling the bungee cord's mechanics. Overall, the results made theoretical sense and were acceptable.

CONCLUSION:

Experimental Outcomes:

- This experiment displayed that the k value was not constant for the bungee cord when the force on the bungee was varied
- Despite the variation in the k value, we can still potentially predict the bungee's behavior using Hooke's Law with a modification in the form of a y-intercept

Implications of these conclusions:

- Going forward the best way to model the behavior of the bungee in terms of spring mechanics would be to use the adapted Hooke's Law equation from the linear best fit trendline, $W = 3.08(X_L – X_o) + 0.41$