Relationship between Spring Constant and Length of Bungee Cord

Introduction

In this experiment, we aimed to model the behavior of the bungee cord that will be used in the Bungee Challenge. Specifically, we investigated the relationship between the "spring constant" of the bungee cord and the length of the bungee. Bungee cord stretch behavior is similar to spring like behavior, so we used Hooke's Law in order to determine the measurement for spring constant (k). Hooke's Law (Equation 1) states that the force needed to extend a spring by a certain distance is equal to a spring constant times that distance. The force acting on the bungee cord can be explained by Newton's second law (Equation 2), which states that the force is equal to mass times acceleration. In this static experiment the acceleration is equal to the acceleration due to gravity (g) and the mass is equivalent to the hanging mass.

Equation 1: $F_{spring} = -kx$

Where F is the force of the spring, k is the spring constant and x is the displacement of the spring.

Equation 2: $F = m\mathrm{g}$

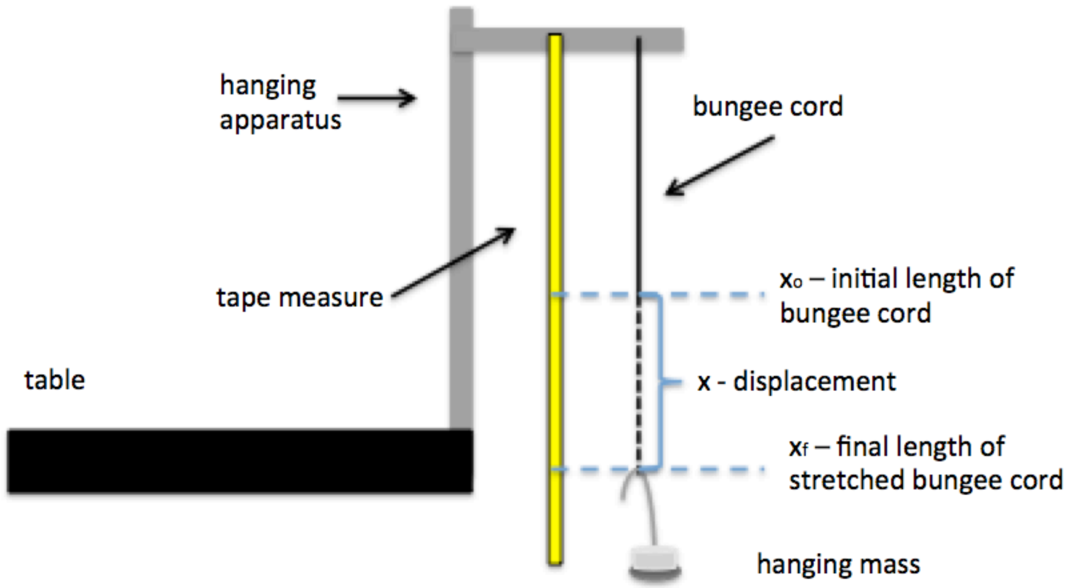
Where F is the force, m is the mass added to the spring and g is the gravitational constant.

Methods

We measured the displacement of the bungee at various mass increments and lengths of cord in order to determine the relationship between spring constant and bungee cord length. This knowledge will allow us to determine the length of bungee cord to use in the Bungee Challenge to meet a certain spring constant in order to ensure a successful egg bungee jump.

We used varying lengths of a single strand of bungee cord (0.20—0.50 m) and a range of masses hanging from the cord at each length (0.15—0.19 kg) to find a spring constant, k, for each length. We tied a knot at one end of the bungee cord and hung it on a screw connected to a metal hanging apparatus attached to the table by a clamp. A measuring tape rested on the hanger in order to measure the length of bungee cord and displacement. We tied another knot at the length of bungee that was being tested to which we added the hanging mass in each trial. Various masses were added and the total length of stretch of the bungee cord was measured. The displacement, x, at each weight was calculated by subtracting the resting length of the bungee, x_0, from the total length of stretched bungeed, x_f. The final length of stretched bungee was measured from the top of the knot tied at the end with the hanging mass. This process was repeated for all five lengths of cord with a single strand of bungee and again at all five lengths with a double stranded bungee. The set up with materials can be seen in Figure 1.

Figure 1: Set up of experiment.



Results

From our data, we found a linear relationship between the force applied to the bungee cord by the mass added, and the displacement, which yielded the spring constant for each length of cord. The spring constant for each length of cord is equivalent to the slope of the corresponding line, as seen in Figures 2 and 3.

Table 1: Displacement of Masses at Varying Lengths for Single Stranded Bungee. Displacements measured from resting length of single stranded cord with hanging masses ranging from 0.15—0.19 g for all five lengths of cord tested.

Cord Length x_0 (m, ± 0.01 m)	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**
Force F (N) F = mg	**Displacement at Each Length** x (m, ± 0.01 m) $x = x_f - x_0$				
1.47	0.21	0.24	0.30	0.41	0.50
1.52	0.22	0.26	0.32	0.43	0.53
1.57	0.23	0.27	0.34	0.46	0.56
1.62	0.24	0.29	0.36	0.48	0.59
1.67	0.26	0.30	0.38	0.53	0.62
1.72	0.27	0.32	0.39	0.54	0.65
1.77	0.28	0.33	0.41	0.56	0.68
1.81	0.29	0.34	0.43	0.57	0.71
1.86	0.30	0.36	0.45	0.61	0.73

Figure 2: Force vs. Displacement for Single Stranded Bungee. Linear relationship between displacement of single stranded bungee cord in response to force acting upon it with equations shown for each length of cord tested. Spring constant (*k*) for each length of cord is equal to the slope of the line that corresponds to that length as indicated by the legend on the right.

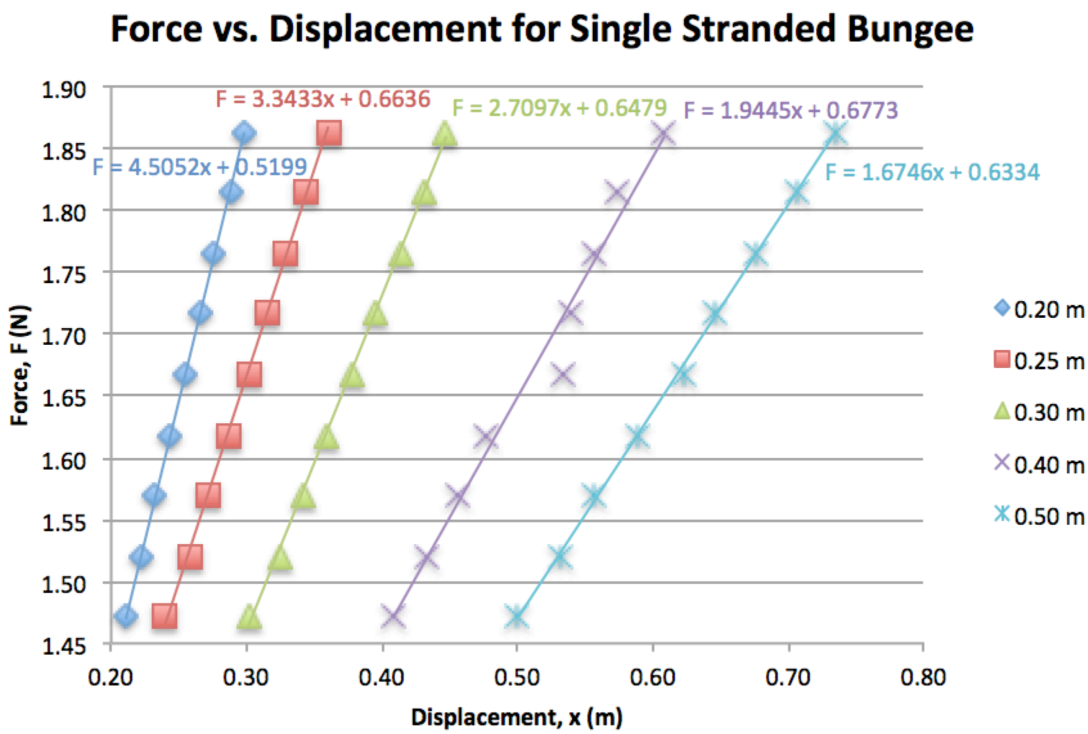
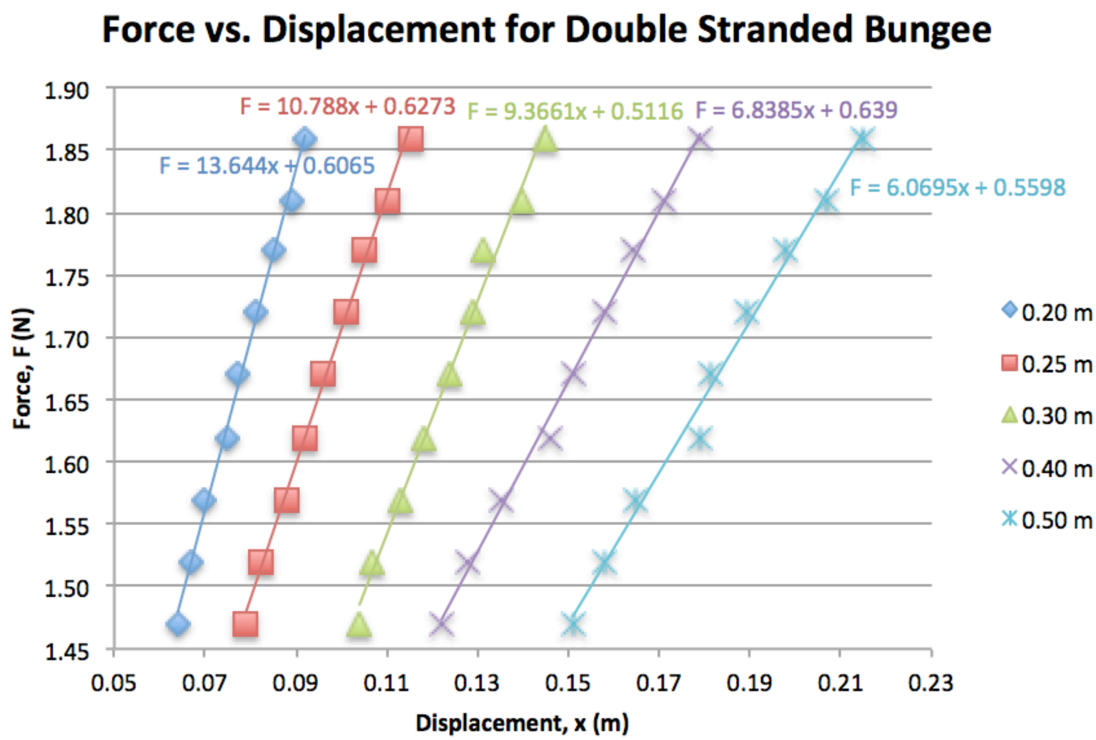


Table 2: Displacement of Masses at Varying Lengths for Double Stranded Bungee. Displacements measured from resting length of double stranded cord with hanging masses ranging from 0.15—0.19 g for all five lengths of cord tested.

Cord Length x_0 (m, ± 0.01 m)	**0.20 m**	**0.25 m**	**0.30 m**	**0.40 m**	**0.50 m**
Force *F* (N) F = mg	**Displacement** *x* (m, ± 0.01 m) $x = x_f - x_0$				
1.47	0.06	0.08	0.10	0.12	0.15
1.52	0.07	0.08	0.11	0.13	0.16
1.57	0.07	0.09	0.11	0.14	0.17
1.62	0.08	0.09	0.12	0.15	0.18
1.67	0.08	0.10	0.12	0.15	0.18
1.72	0.08	0.10	0.13	0.16	0.19
1.77	0.09	0.11	0.13	0.16	0.20
1.81	0.09	0.11	0.14	0.17	0.21
1.86	0.09	0.12	0.15	0.18	0.22

Figure 3: Force vs. Displacement for Double Stranded Bungee. Linear relationship between displacement of double stranded bungee cord in response to force acting upon it with equations shown for each length of cord tested. Spring constant (*k*) for each length of cord is equal to the slope of the line that corresponds to that length as indicated by the legend on the right.

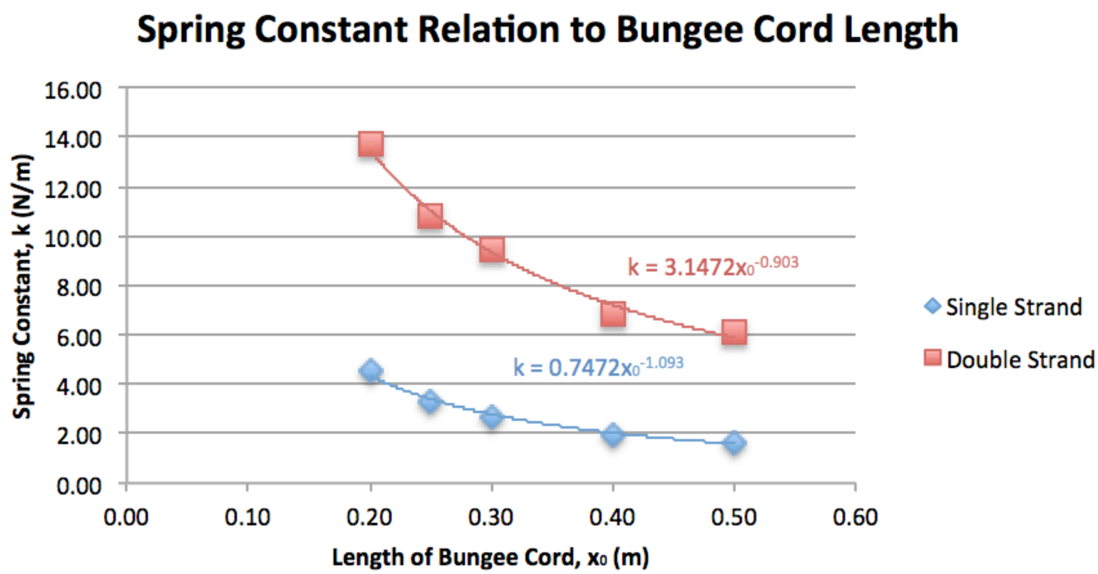


We then plotted the spring constants (k) for both single and double stranded bungee cords against their respective lengths of cord and found a power relationship that describes the spring constant in terms of cord length (Figure 4).

Table 3: Spring Constants for Each Length of Bungee Tested. The spring constants for single and double stranded bungee cords derived from the linear equations produced in Figures 2 and 3.

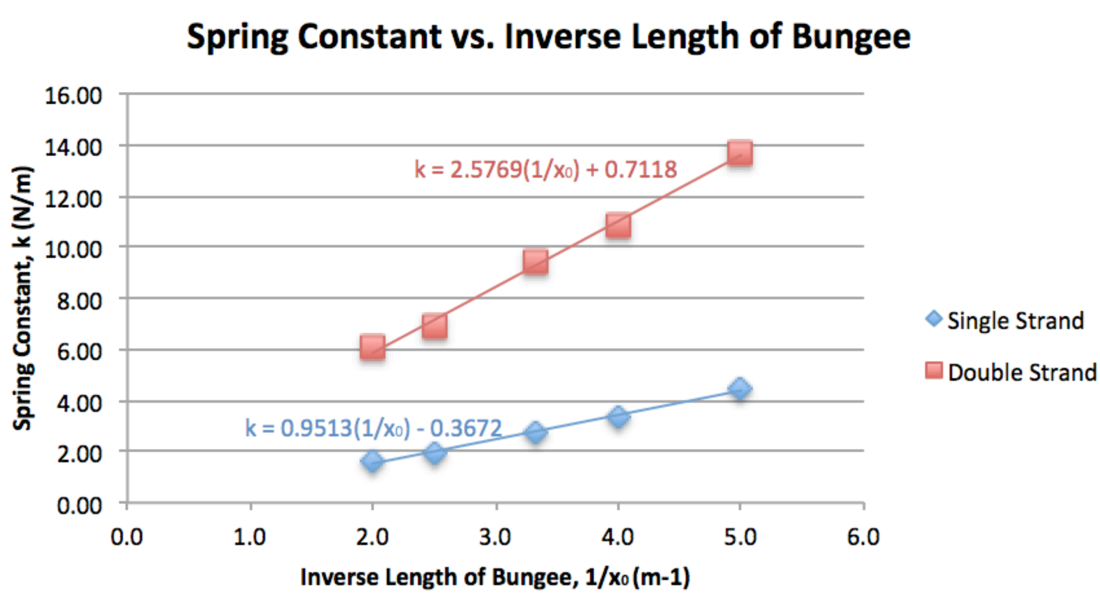
Length of Bungee Cord *x* (m, ± 0.01 m)	**Spring Constant** *k (N/m, ± 0.01 N/m)* **Single Strand**	**Double Strand**
0.20	4.51	13.71
0.25	3.34	10.84
0.30	2.71	9.42
0.40	1.94	6.87
0.50	1.67	6.10

Figure 4: Spring Constant Relation to Bungee Cord Length. The relationship between spring constant, *k*, and length of bungee cord, x_0, can be see for both single and double stranded bungee cords in the power functions displayed below. As the length of the bungee cord increases, the spring constant decreases at a rate of 1.093 for the single stranded bungee and 0.903 for the double stranded bungee.



The data from Figure 4 was manipulated by plotting the spring constant against the inverse of the length of bungee cord ($1/x_o$) in order to produce a linear relationship between these two values (Figure 5).

Figure 5: Spring Constant vs. Inverse Length of Bungee. The linear relationship displayed between the spring constant, *k*, and the inverse of the length of bungee ($1/x_0$) shows that the length of cord directly impacts the spring constant.



The slopes of the equations shown in Figure 5 are equal to the proportionality constant that relates the spring constant (k) to the length of bungee cord (x_0). This means that the spring constant of a particular bungee cord can be determined for a given length of either single stranded (Equation 3) or double stranded (Equation 4) bungee cord.

Equation 3: $k = 0.9513(1/x_0) - 0.3672$

Equation 4: $k = 2.5769\ (1/x_0) + 0.7118$

Linear regression analysis of this data determined the standard error of the slope of these functions to be 0.06 for the single stranded bungee cord and 0.11 for the double stranded bungee cord. The uncertainty of the intercept is 0.20 and 0.38 for the single and double stranded cords, respectively.

Discussion

This experiment proves that the behavior of bungee cord is not the same as ideal spring behavior. The relationship between k value and displacement is linear, as seen in Figures 2 and 3 and this relationship is in accordance with Hooke's law (Equation 1). The spring constant of bungee cord is variable depending on the length of the cord. The different coefficients for each line corresponding to a different length of bungee cord in Figures 2 and 3 provide proof of this. As the length of the bungee cord increases, the k value decreases, meaning that the same net force acting on the bungee cord will result in a larger displacement when applied to the longer bungee cord. Increasing the length of the bungee cord is similar to adding two springs together in series and getting a resulting k value that is smaller than the k value of the individual springs alone.

This experiment also investigated the behavior of single stranded bungee cord versus double stranded bungee cord at the same lengths and with the same forces acting upon them. Doubling the bungee cord is similar to adding two springs together in parallel and getting a resulting k value that is larger for the same force applied to a constant length of cord. This is evident in the larger k values (slopes) for the lines in Figure 3 compared to those in Figure 2. The data for double stranded bungee cords still supports the linear relationship modeled between k value and displacement, as discussed above.

Despite the linear relationship between k value and displacement, the data collected is not in accordance with Hooke's law. This is due to the y-intercept for each of the lines in Figures 2 and 3. This value corresponds to the force acting upon the bungee cord when the displacement is zero. According to Hooke's law, an ideal spring at zero displacement will have a net force that is also equal to zero. The y-intercept for each equation in Figures 2 and 3 shows that Hooke's law does not hold true in the case of bungee cord. Bungee cord behaves differently than ideal springs, as seen through this net force acting upon the spring even at displacement equal to zero. This is due to the fact that elastic string behaves differently at small stretches compared to large ones. It can be noted that this present force acting upon the bungee cord remains relatively constant for varying lengths of cord, and also constant among both single and double stranded bungee cords. The average y-intercept, or force acting on the various lengths of single stranded bungee cord at displacement zero equals 0.63 N, while that of the double stranded bungee cord at displacement zero equals 0.59 N. The differences between these values among each strand were most likely a result of the uncertainties in the investigation that are discussed below.

The raw uncertainties for displacement of the bungee cord (± 0.01 m) relate to the difficulty in determining the displacement with the tape measure. The equilibrium point for each trial was difficult to determine because any slight contact with either the hanging mass or the cord itself caused the bungee cord to oscillate. Also, the cord tended to stretch out more, even over the small time during which the final displacements were recorded. This was much more evident with the single stranded bungee cord at longer lengths.

A possible source of error that could have caused deviations in the data points collected could have resulted from the knots tied at each end of the bungee cord. Tying a knot creates a loop through which to hang the hanger for the masses added to the bottom, or a loop to hang over a screw at the top of the hanging apparatus. Both of these knots are necessary for the experiment, but these loops create a small portion of bungee cord that is doubled, which we know changes the properties of the k value. We did our best to minimize the size of the loop, thus minimizing the effect of this doubled portion of the bungee cord, but regardless, this error could have resulted in slight variations in the k values observed from this data. In the future, this error could be eliminated if we found a way to attach the free weights to the bottom of the bungee without having to double the cord, or reduced if the same knot was kept in the end of the cord to which the hanger was added onto for each trial.

Figure 5 displays the relationship between the spring constant (k) and the inverse length of bungee cord ($1/x_0$). A linear regression analysis was produced for this data, which determined the statistical uncertainties for the k values and inverse lengths for both single and double stranded bungee cords. The standard error of the slope, referring to the inverse length of bungee cord, for the single stranded bungee was 0.06, a relatively small value. The standard error of the slope for the double stranded bungee cord was 0.11. The sources of uncertainty in measurement of the length of bungee cord and displacement lengths contributed to this statistical error and tell us that the precision of the displacement values should be considered to two decimal places for the single stranded bungee and one decimal place for the double stranded bungee.

The standard error of the intercepts refers to the force acting on the bungee cord at its initial displacement of zero. This uncertainty was larger than that of the slope for both single and double stranded bungee cords. For the single stranded bungee, the uncertainty of the intercept was 0.20, while the uncertainty for the double stranded bungee was 0.38. This large uncertainty for the intercepts tells us that this intercept is not a precise value, which can largely be attributed to the various sources of uncertainty and error described previously.

To minimize this uncertainty in the future, measures can be taken to more accurately record the resting and displacement lengths of bungee cord, as well as tying one small knot to the end of the bungee with the hanger attached that will remain the same for all measurements of the single stranded bungee in order to provide more consistency between trials at different lengths.

Conclusion

This experiment aimed to model the relationship between the "spring constant" of a bungee cord in relation to the cord length. Despite the difference in behavior between bungee cord and ideal springs, we ultimately found an equation in which the length of the bungee cord can be substituted in for either a single or double stranded bungee cord, and the spring constant (k value) can be predicted. By examining the power relationship between various lengths of single and double stranded bungee cord, previous findings from past investigations regarding the relationship between the k value and number of strands of bungee (single, double, triple, etc.) were confirmed. Both of these findings can be combined in the Bungee Challenge in order to determine the length of a single, double, or triple stranded bungee cord needed to achieve a particular k value for a successful and thrilling egg bungee drop.