Can Hooke's Law be used to model the stretch of a bungee cord?

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Introduction

If you want to experience an adrenaline rush while understanding physics, look no further than bungee jumping. Our purpose is to give a raw egg the ride of its life in the Bungee Challenge. The Bungee Challenge involves "dare-deviling" a raw egg using experimentally determined characteristics that will allow the egg to experience a quality bungee jump. A quality bungee jump will be one that allows for the longest possible time of free fall for an egg at a high speed before safe deceleration to avoidance of the ground. To begin solving this problem, we must model the behavior of the cord being used for the bungee challenge. The most important behavior we need to model is the amount of stretch and recoil of the bungee cord, as this will eventually allow us to determine how long we must make the bungee cord for the dare-deviled jump. Hooke's Force Law describes the restoring force of a spring. Because the cord being used for the bungee challenge has stretch and recoil, we assumed that Hooke's Force Law could model these properties. Hooke's force law equation states that the force of the spring is equivalent to the product of the spring constant and the change in length of the spring. The Hooke's force law equation is as follows:

$$\mathbf{F} = -kx \tag{1}$$

Where **F** is the spring force, k is the spring constant, and x is the change in length of the spring.

The overarching goal of our experiment is to model a bungee jump using force laws and to test it using an egg in a harness attached to an elastic "bungee" cord. In this first week, our primary experiment was to model the spring constant of the elastic cord by determining the relationship between the change in length of the elastic string and the spring constant. To achieve this, we varied the mass of hanging weights on the cord and measured the displacement of the elastic to obtain the value of our spring constant.

Methods

We aimed to find the spring constant of our bungee cord. To do this, we tried to determine the relationship between displacement of the cord and the weight of the object that was hung on the cord. We used the displacement of the cord as the x in equation 1 and the weight of the hanging mass as F. The elastic was stretched and knotted at two ends. It was then placed on the stand and its unstretched length was measured (setup depicted in Figure 1). We made this unstretched length the equilibrium point. We then placed a hanger of mass 0.05 kg on the cord, measured the cord, and recorded the displacement of the hanger. We repeated this ten times, each time adding 0.01 kg more of mass onto the hanger. To determine if the spring constant differed with the length of the unstretched cord, we completed three trials following this procedure: one trial using 0.205 m of cord, one trial using 0.410 m of cord, and one trial using 0.640 m of cord.

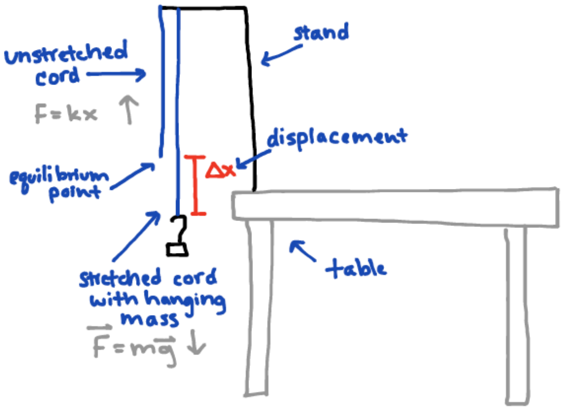


Figure 1. Setup for all three trials of our experiment. A stand was placed on a table. The cord was attached to the stand and a hanging mass was placed upon it. The upward force acting on the hanging mass is denoted **F** = kx, while the downward force acting on the hanging mass is denoted **F** = mg.

Results

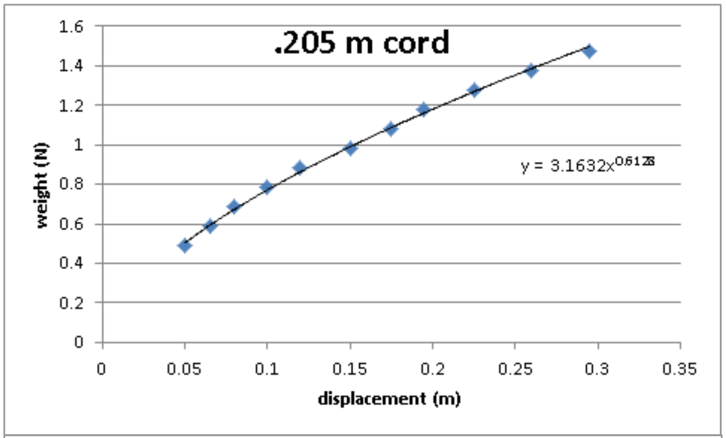
We found the spring constant of the elastic cord using our setup, graphs, and equation 1. We used the weight of the hanging mass (mg) as the spring force, and set that equal to the spring force times the displacement of the mass. From this, we were able to solve for the spring constant of our elastic. Our raw data for displacement of each length of cord is below.

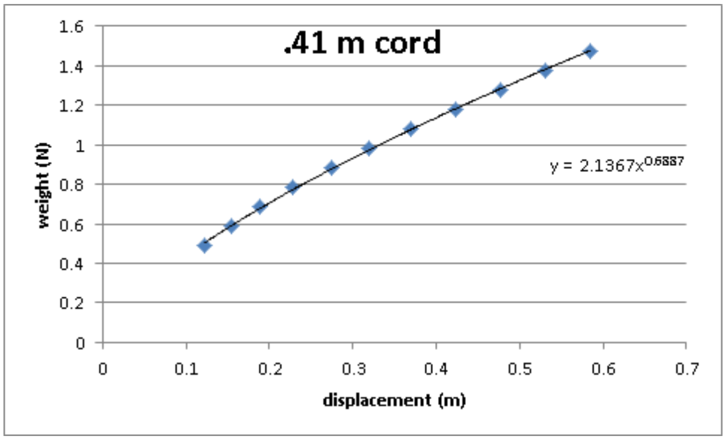
0.205 m unstretched cord	
Hanging mass (g)	**Displacement (± 0.1 cm)**
0	0
50	5
60	6.5
70	8
80	10
90	12
100	15
110	17.5
120	19.5
130	22.5
140	26
150	29.5

0.41 m unstretched cord	
Hanging mass (g)	**Displacement (± 0.1 cm)**
0	0
50	12.2
60	15.4
70	18.8
80	22.8
90	27.5
100	32
110	37
120	42.3
130	47.7
140	53
150	58.5

0.64 m unstretched cord	
Hanging mass (g)	**Displacement (± 0.1 cm)**
0	0
50	17.3
60	21.8
70	27
80	33
90	39.5
100	46.5
110	54.5
120	63.5
130	72.5
140	80.5
150	89

Table 1. Stretch of the elastic cord is related to displacement of a hanging mass. Raw data for the displacements of three lengths of elastic cords with varying masses.





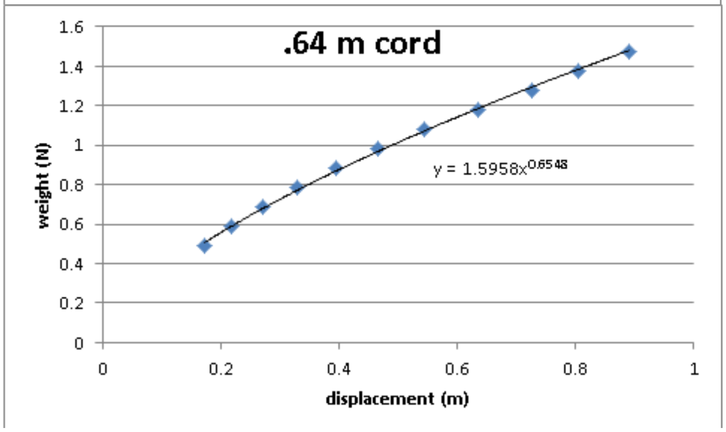


Figure 2. Relationship of displacement of the cord to the weight of the hanging mass. The x axis refers to the displacement of the cord, while the y axis refers to the weight of the hanging mass, or the force on the system.

Figure 2 shows the relationship we found between the displacement of the cord to the weight of the hanging mass. To obtain this graph, we graphed our raw data in Excel and fit it to the most accurate trend function. For our data, this was a power function. We then took the power function relationship and determined that our spring constant k was the coefficient of the x term in each of the power functions (Table 2).

Cord length (± 0.0001 m)	K (± 0.00001 N/m)
.205	3.1632
.41	2.1367
.64	1.5958

Table 2. Relationship between cord length and the spring constant of the cord. This was obtained from the coefficients of the x term in the power function obtained from each graph.

Discussion

We determined that the relationship between the spring constant and the stretch of the elastic was a power function, rather than the simple linear spring constant seen in equation 1. We believe that the coeffiecient of x in our power function equations equals the spring constant for the bungee cord. The y-intercept values that we obtained did not equal zero, which seems to say that there is a force acting on the bungee cord even when there is no mass added to the elastic. Furthermore, the intercepts differed with the length of the cord. This could be a reflection of the error of our measurements, or could be a reflection of some force that we need to take into account in our calculations. We believe that we have modeled the spring constant of the elastic using changes in the displacement of the elastic.

Conclusion

The goal of our experiment was to determine the spring constant of the elastic cord, which we accomplished. We found that the recoil of the spring differs with the length of the cord used, which will have to be taken into account during the final bungee challenge. For instance, we could use either all elastic cord to act as our bungee, or we could add a piece of non-stretching cord. In the first instance, the cord would have a small spring constant, which would ensure that the egg comes close to the ground, but may bring it too close to the ground so that it cracks. In the second instance, the cord would have a larger spring constant, which would prevent the egg from coming too close to the ground; however, one of the qualities of a good bungee jump is a long time of free fall. Our preliminary results show that we should probably use a long length of cord to ensure a long time of free fall, but accurately measure the cord so that the egg does not hit the ground. This is the first piece of the puzzle in our end-of-term experiment. Our overarching goal of designing a quality bungee jump for a raw egg is still to be achieved. In later experiments, we will be able to use this force constant to ensure that our daredeviled egg lives through our test.